

Date: 7-03-2022

Registration number:

ST. JOSEPH’S COLLEGE (AUTONOMOUS), BENGALURU-27

M.Sc. Statistics - I SEMESTER

SEMESTER EXAMINATION: OCTOBER 2021

(Examination conducted in January-March 2022)

**ST 7421 – Mathematical Analysis and Linear Algebra**

 Time- 2 ½ hrs Max Marks-70

This question paper contains TWO printed pages and TWO parts

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**Part A**

**Answer any 6 questions 6X3=18**

1. Define Cauchy criteria for convergence of a series.
2. Give two properties of R-S integrals?
3. Define Weierstrass-M test.
4. State Leibnitz rule.
5. Distinguish Pointwise and uniform convergence
6. State Caley Hamilton theorem.
7. Define Vector spaces and subspaces.
8. Explain Moore Penrose inverse of a matrix.

**Part B**

**Answer any 4 questions 4X13=52**

1. A) State and prove fundamental theorem of integral calculus. (6)

B) If is continuous on [a, b] and α is monotonic function on [a, b], show that $f\in R \left(α\right)$. (7)

1. A) Define Beta and Gamma integrals and their properties (6)
2. Prove that If p is a limit point of a set E, then every neighborhood of p contains infinitely many points of E. (7)
3. A) Find the maximum value of the function $f\left(x\right)= x^{2}-4xy-6y^{2}$. (6)

B) Prove that If $\{fn\}$ is a sequence of continuous functions on E and if $fn\rightarrow f $converges uniformly on E, then f is continuous on E. (7)

1. A) State and prove Dirichlet’s test for the convergence of improper integrals. (7)

B) Prove that $\sum\_{}^{}\frac{1}{n^{p}}$ Converges if p>1 and diverges if $p\leq 1.$ (6)

1. A) Eshtablish the equivalence relation and congruence of a matrices and set of all nXn matrices over field $F.$ (6)

B) Prove That $f\in R \left(α\right)$ on [a,b], m$\leq f\leq M, ϕ$ is continuous on [m, M] and $h\left(x\right)= ϕ(f\left(x\right))$ on [a, b], then $h\in R(x)$ on [a, b] (7)

1. A) Discuss the orthonormality of a matrix and diagonalization of a matrix. (6)

B) If two functions f, g defined on [a, b] are

* 1. Continuous on [a, b],
	2. Derivable on ]a, b[ and
	3. g’(x) /= 0, for any x$ \in $ ]a, b[

Then there exists one real number c between a and b such that (7)

$$\frac{f\left(b\right)-f(a)}{g\left(b\right)-g(a)}= \frac{f'(c)}{g^{'}(c)}$$